B.Sc(H)-Math(?) [illegible] comparison test ①

Paper-III Unit-B Q. Test the series $\frac{1}{2\cdot3\cdot4}+\frac{[illegible]}{4\cdot5\cdot6}+\frac{[illegible]}{6\cdot7\cdot8}+\dots$

Infinite series

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[illegible]

Here $u_n = \frac{[illegible]}{2n(2n+1)(2n+2)} = \frac{[illegible]}{[illegible]}$

$= \frac{[illegible]}{2n^3(1+\frac{1}{2n})(1+\frac{1}{n})}$

Let us compare by an auxiliary series whose nth term $v_n = \frac{1}{n^2}$

$\therefore \frac{u_n}{v_n} = \frac{[illegible]}{2(1+\frac{1}{2n})(1+\frac{1}{n})}$

$\lim_{n\to\infty} \frac{u_n}{v_n} = \lim_{n\to\infty} \frac{1}{2(1+\frac{1}{n})(1+\frac{1}{2n})} = \frac{1}{2}$ [illegible]

Hence $\frac{u_n}{v_n}$ is finite. But the series $\sum v_n$ whose nth term is $\frac{1}{n^2}$ is convergent ($\because p>1$). So the given series by comparison test is also convergent.

② Test the series whose nth term is

$\sqrt{[illegible]} - \sqrt{[illegible]}$

Sol. Here $u_n = \sqrt{[illegible]} - \sqrt{[illegible]}$

$= \frac{(\sqrt{[illegible]} - \sqrt{[illegible]})(\sqrt{[illegible]} + \sqrt{[illegible]})}{\sqrt{[illegible]} + \sqrt{[illegible]}}$

$$\frac{(n+1)-(n-1)}{\sqrt{n+1}+\sqrt{n-1}} = \frac{2}{\sqrt{n+1}+\sqrt{n-1}}$$

$$= \frac{2}{\sqrt{n}\left\{\sqrt{1+1/n}+\sqrt{1-1/n}\right\}}$$

Let us compare the given series $\sum u_n$ with the auxiliary series $\sum v_n$ whose nth term $= \frac{1}{\sqrt{n}}$

$$\therefore \frac{u_n}{v_n} = \frac{2}{\left(\sqrt{1+1/n}+\sqrt{1-1/n}\right)}$$

$$\therefore \lim_{n\to\infty} \frac{u_n}{v_n} = \lim_{n\to\infty} \frac{2}{\sqrt{1+1/n}+\sqrt{1-1/n}} = \frac{2}{2} = 1 = \text{a finite quantity}$$

Hence $\frac{u_n}{v_n}$ is finite. But the auxiliary series whose nth term is $\frac{1}{\sqrt{n}}$ is divergent ($\because p = 1/2 < 1$). So the given series $\sum u_n$ is also divergent.